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Question Paper Code : 20815

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2022.

Fourth Semester

Computer Science and Engineering

MA 8402 — PROBABILITY AND QUEUEING THEORY

(Regulations 2017)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. What are the limitations of Binomial distribution?

2. A fair coin was tossed two times. Given that the first toss resulted in heads, what is the probability that both tosses resulted in heads?

3. The joint pdf of a two-dimensional random variable (X,Y) is given by

$$f(x,y) = \begin{cases} ke^{(x+y)} & ; 0 \le x \le y,\ 0 \le y \le \infty \\ 0 & ; otherwise \end{cases}$$. Find the value of 'k'.

4. In a partially destroyed laboratory, a record of an analysis of correlated data, the following results only are legible: Variance of $X = 9$; Regression equations are $8X - 10Y + 66 = 0$ and $40X - 18Y = 214$. What are the mean values of X and Y?

5. Is Poisson process stationary? Justify.

6. What do you mean by $X_n = 6$ and P_{ij} in Markov chain?

7. What do you mean by steady state of a queueing system?

8. Patients arrive at the Government hospital for emergency service at the rate of one per hour. Currently only one emergency case can be handled at a time. Patients spend on an average of 20 mins receiving emergency care. Find the probability that a patient arriving at the hospital will have to wait.

9. What do you mean by Non-Markovian queueing models?

10. State Jackson's theorem for an open network.

PART B — (5 × 16 = 80 marks)

11. (a) (i) A student buys 1000 integrated circuits (ICs) from supplier A, 2000 ICs from supplier B, and 3000 ICs from supplier c. He tested the ICs and found that the conditional probability of and IC being defective depends on the supplier from whom it was bought. Specifically, given that an IC came from supplier A, the probability that it is defective is 0.05; given that an IC came from supplier B, the probability that it is defective is 0.10; and given that an IC came from supplier C, the probability that it is defective is 0.10. If the ICs from the three suppliers are mixed together and one is selected at random, what is the probability that it is defective? (8)

(ii) The CDF of the random variable X is given by

$$F_X(x) = \begin{cases} 0 & ; x < 0 \\ x + \frac{1}{2} & ; 0 \le x \le \frac{1}{2} \\ 1 & ; x > \frac{1}{2} \end{cases}$$ Using CDF, compute $P\left(X > \frac{1}{4}\right)$ and

$P\left(\frac{1}{3} < X \le \frac{1}{2}\right)$ (8)

Or

(b) (i) Messages arrive at a switchboard in a Poisson manner at an average rate of six per hour. Find the probability for each of the following events: (1) exactly two messages arrive within one hour (2) no message arrives within one hour (3) at least three messages arrive within one hour. (8)

(ii) The weights in pounds of parcels arriving at a package delivery company's warehouse can be modelled by an N(5,16) normal random variable X. (1) What is the probability that a randomly selected parcel weighs between 1 and 10 pounds? (2) What is the probability that a randomly selected parcel weighs more than 9 pounds? (8)

12. (a) For the bivariate probability distribution of (X,Y) given below, find $P(X \le 1), P(Y \le 3), P(X \le 1, Y \le 3)$, $P(X \le 1 / Y \le 3)$, $P(Y \le 3 / X \le 1)$ and $P(X + Y \le 4)$. (16)

X \ Y	1	2	3	4	5	6
0	0	0	1/32	2/32	2/32	3/32
1	1/16	1/16	1/8	1/8	1/8	1/8
2	1/32	1/32	1/64	1/64	0	2/64

Or

(b) (i) Find the coefficient of correlation between X and Y, using following data: (8)

X	65	66	67	67	68	69	70	72
Y	67	68	65	68	72	72	69	71

(ii) If $X_1, X_2, X_3, \ldots X_n$ are Poisson variates with parameter $\lambda = 2$, Use the central limit theorem to estimate $P(120 \le S_n \le 160)$, where $S_n = X_1 + X_2 + X_3 +, \ldots + X_n$ and $n = 75$. (8)

13. (a) (i) Suppose that customers arrive at a bank, according to a Poisson process with a mean rate of 3 per minute; find the probability that during a time interval of 2 min. (1) exactly 4 customers arrive and (2) more than 4 customers arrive. (8)

(ii) Three boys A,B and C are throwing ball to each other. A always throws the ball to B and B always throws the ball to C, but C is just as likely to throw the ball to B as to A. Show that the process is Markovian. Find the transition Matrix and also find P^2 (8)

Or

(b) (i) An electrical engineer analyzing a series of digital signals generated by a testing system observes that only one out of 15 highly distorted signals follow a highly distorted signal with no recognizable signal between, whereas 20 out of 23 recognizable signals follow recognizable signal with no highly distorted signals between. Given that only highly distorted signals are not recognizable, find the fraction of signals that are highly distorted. (8)

(ii) A fair die is tossed repeatedly. The maximum of the first n outcomes is denoted by X_n. Is $\{X_n : n = 1,2,\ldots\}$ a Markov chain? Why or why not? If it is a Markov chain, calculate its transition probability matrix. (8)

14. (a) Explain Markovian Birth - Death process and obtain the expressions for steady state probabilities.

Or

(b) In a railway marshalling yard, goods trains arrive at a rate of 30 trains per day. Assuming that the inter-arrival time follows an exponential distribution and the service time is also exponential with an average of 36 mins. Calculate the following:

(i) The mean queue size

(ii) The average number of trains in the queue

(iii) The probability that the queue size exceeds 10

(iv) If the input of trains increases to an average 33 per day, what will be the change in (i) and (iii).

15. (a) Derive Pollaczek - Khintchine formula of an M/G/1 queue.

Or

(b) Write short notes on (i) open queueing networks and (ii) closed queueing networks.

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